

# Beam normal spin asymmetry in elastic lepton-nucleon scattering

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## Abstract

We discuss the two-photon exchange contribution to observables which involve lepton helicity flip in elastic lepton-nucleon scattering. This contribution is accessed through the spin asymmetry for a lepton beam polarized normal to the scattering plane. We estimate this beam normal spin asymmetry at large momentum transfer using a parton model and we express the corresponding amplitude in terms of generalized parton distributions.

*Key words:* Elastic electron nucleon scattering, perturbative calculations, generalized parton distributions

*PACS:* 25.30.Bf, 12.38.Bx, 13.40.Gp

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## 1 Introduction

Elastic electron-nucleon scattering in the one-photon exchange approximation gives direct access to the electromagnetic form factors of the nucleon, an essential piece of information about its structure. In recent years, the ratio  $G_{Ep}/G_{Mp}$  of the proton's electric to magnetic form factors has been measured up to large momentum transfer  $Q^2$  in precision experiments [1,2] using the

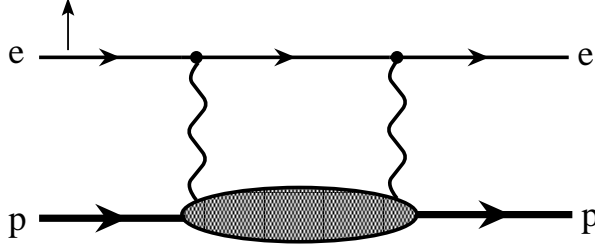


Fig. 1. Two-photon exchange amplitude entering the elastic lepton-nucleon scattering, with a beam spin polarized normal to the scattering plane.

polarisation transfer method. It came as a surprise that these experiments for  $Q^2$  up to  $5.6 \text{ GeV}^2$  extracted a ratio of  $G_{Ep}/G_{Mp}$  which is incompatible with unpolarized measurements [3,4,5] using the Rosenbluth separation technique. It has been suggested on general grounds in Ref. [6] that this puzzle may be explained by a two-photon exchange amplitude (see Fig. 1) whose magnitude is a few percent of the one photon exchange term. The failure of the one-photon approximation for elastic electron-nucleon scattering is amplified at large  $Q^2$  in the case of the Rosenbluth extraction of  $G_{Ep}/G_{Mp}$ , but affects the polarization method only little. A model calculation of the two-photon exchange amplitude when the hadronic intermediate state is a nucleon was performed in Ref. [7]. It found that the  $2\gamma$  exchange correction with intermediate nucleon can partially resolve the discrepancy between the two experimental techniques. Very recently, the two-photon exchange contribution to elastic electron-nucleon scattering has been estimated at large momentum transfer [8], through the scattering off a parton in the proton by relating the process on the nucleon to the generalized parton distributions. This calculation found that the  $2\gamma$  exchange contribution is indeed able to quantitatively resolve the discrepancy between Rosenbluth and polarization transfer experiments.

To use electron scattering as a precision tool, one needs a good control of the two-photon exchange mechanisms, which justifies a systematic study of these effects, both theoretically and experimentally. The real (dispersive) part of the two-photon exchange amplitude can be accessed through the difference between elastic electron and positron scattering off a nucleon. The imaginary (absorptive) part of the two-photon exchange amplitude on the other hand can be accessed through a single spin asymmetry (SSA) in elastic electron-nucleon scattering, when either the target or beam spin are polarized normal to the scattering plane. As time reversal invariance forces this SSA to vanish in the one-photon exchange approximation, it requires the exchange of at least two photons between lepton and nucleon (see Fig. 1), and hence it is of order  $\alpha = e^2/(4\pi) \simeq 1/137$ .

In the context of QED, such normal spin asymmetries have been discussed

long time ago [9]. In Ref. [9], the leading order (box diagram) calculations for  $e^{-\uparrow}\mu^{-}$  and  $e^{-\uparrow}e^{-}$  (Møller) elastic scattering with an electron polarized normal to the scattering plane have been performed. The resulting SSA is directly sensitive to a QED rescattering phase. Recently, the E158 experiment at SLAC [10] has measured the Møller scattering  $e^{-\uparrow}e^{-} \rightarrow e^{-}e^{-}$  with normal beam spin polarization, allowing for a precision comparison with the latest QED calculations [11], including leading logarithmic QED corrections due to initial and final state radiation effects.

To get an order of magnitude estimate for the beam normal SSA, we have to consider that, besides the reduction by an overall factor  $\alpha$ , the polarization of an ultra-relativistic electron in the direction normal to its momentum involves a suppression factor  $m_e/E_e$  (with  $E_e$  the electron beam energy), which is of order  $10^{-4}$  to  $10^{-3}$  for beam energies in the few GeV range. Therefore, the beam normal SSA is of order  $10^{-6}$  (for some order of magnitude estimates, see Refs. [12,13]). A measurement of such small asymmetries is quite demanding experimentally. However, in the case of a polarized lepton beam, asymmetries of the order ppm are currently accessible in parity violation (PV) elastic electron-nucleon scattering experiments. The parity violating asymmetry involves a beam spin aligned in the direction of its momentum. The SSA for an electron beam spin normal to the scattering plane, which corresponds with a flip of the lepton helicity, can also be measured using the same experimental set-ups. First measurements of this beam normal SSA at beam energies below 1 GeV have yielded values as large as 10 ppm in magnitude [14,15,10]. At higher beam energies, the beam normal SSA can be measured in upcoming PV elastic electron-nucleon scattering experiments [16,17]. One may therefore envisage the possibility in near future to access this asymmetry at larger momentum transfer (for  $Q^2$  around or larger than  $1 \text{ GeV}^2$ ), where one may expect that the scattering off a parton starts to dominate.

The aim of the present work is to develop the formalism for elastic electron-nucleon scattering with a flip of the lepton helicity. To provide an estimate for the two-photon amplitudes, we extend the partonic calculation of Ref. [8]. In this partonic calculation, the real and imaginary parts of the  $2\gamma$  exchange amplitudes are related. Hence in the partonic regime, a direct comparison of the imaginary part with experiment can also provide a very valuable cross-check on the calculated result for the real part.

In Section 2, we develop the formalism for elastic lepton-nucleon scattering which involves a flip of the lepton helicity. In Section 3, we calculate the two-photon exchange lepton-quark scattering amplitude. We then construct in Section 4 the lepton helicity flip amplitudes for the nucleon through a convolution of the quark amplitudes with a generalized parton distribution. We discuss our results in Section 5 and conclude in Section 6.

## 2 Elastic lepton-nucleon scattering formalism beyond one-photon exchange

In this work, we consider the elastic lepton-nucleon scattering :

$$l(k) + N(p) \rightarrow l(k') + N(p'), \quad (1)$$

for which we adopt the definitions :

$$P = \frac{p + p'}{2}, \quad K = \frac{k + k'}{2}, \quad q = k - k' = p' - p, \quad (2)$$

and choose  $Q^2 = -q^2$  and  $\nu = K.P$  as the independent invariants of the scattering. They are related to the Mandelstam invariants  $s = (k + p)^2$  and  $u = (k' - p)^2$  through :  $s - u = 4\nu$  and  $s + u = Q^2 + 2M^2$ , where  $M$  is the nucleon mass. For further use, we also introduce the usual polarization parameter  $\varepsilon$  of the virtual photon, which can be related to the invariants  $\nu$  and  $Q^2$  as (neglecting the lepton mass  $m_l$ ) :

$$\varepsilon = \frac{\nu^2 - M^4 \tau (1 + \tau)}{\nu^2 + M^4 \tau (1 + \tau)}, \quad (3)$$

with  $\tau = Q^2/(4M^2)$ . For a theory which respects Lorentz, parity and charge conjugation invariance, the general amplitude for elastic scattering of two spin 1/2 particles depends on six invariant amplitudes as given by Goldberger *et al.* [18]. The amplitude can be expanded in the following set of invariants :

$$\begin{aligned} \bar{u}(k')u(k) &\cdot \bar{u}(p')u(p), \\ \bar{u}(k')u(k) &\cdot \bar{u}(p')\gamma.Ku(p), \\ \bar{u}(k')\gamma_5 u(k) &\cdot \bar{u}(p')\gamma_5 u(p), \\ \bar{u}(k')\gamma.Pu(k) &\cdot \bar{u}(p')\gamma.Ku(p), \\ \bar{u}(k')\gamma.Pu(k) &\cdot \bar{u}(p')u(p), \\ \bar{u}(k')\gamma_5\gamma.Pu(k) &\cdot \bar{u}(p')\gamma_5\gamma.Ku(p), \end{aligned} \quad (4)$$

where the last three structures conserve the helicity of the lepton and the first three flip it (i.e. are of the order of the mass of the lepton,  $m_l$ ). Using the Dirac equation and elementary relations among the Dirac matrices, the last structure in Eq. (4) can be traded for  $\bar{u}(k')\gamma_\mu u(k) \cdot \bar{u}(p')\gamma^\mu u(p)$ , and a combination of the first five structures. Therefore, one can write the general elastic lepton-nucleon scattering amplitude as :

$$T = T^{non-flip} + T^{flip}, \quad (5)$$

with [6] :

$$T_{h'\lambda'_N, h\lambda_N}^{non-flip} = \frac{e^2}{Q^2} \bar{u}(k', h') \gamma_\mu u(k, h) \times \bar{u}(p', \lambda'_N) \left( \tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \right) u(p, \lambda_N), \quad (6)$$

and :

$$T_{h'\lambda'_N, h\lambda_N}^{flip} = \frac{e^2}{Q^2} \frac{m_l}{M} \left[ \bar{u}(k', h') u(k, h) \cdot \bar{u}(p', \lambda'_N) \left( \tilde{F}_4 + \tilde{F}_5 \frac{\gamma \cdot K}{M} \right) u(p, \lambda_N) + \tilde{F}_6 \bar{u}(k', h') \gamma_5 u(k, h) \cdot \bar{u}(p', \lambda'_N) \gamma_5 u(p, \lambda_N) \right], \quad (7)$$

where  $h(h')$  are the helicities of the incoming (outgoing) leptons,  $\lambda_N(\lambda'_N)$  are the helicities of the incoming (outgoing) nucleons, and where  $m_l$  denotes the lepton mass. In Eqs. (6) and (7),  $\tilde{G}_M$ ,  $\tilde{F}_2$ ,  $\tilde{F}_3$ ,  $\tilde{F}_4$ ,  $\tilde{F}_5$ ,  $\tilde{F}_6$  are complex functions of  $\nu$  and  $Q^2$ , and the factor  $e^2/Q^2$  has been introduced for convenience, with  $e$  the proton charge. Furthermore in Eq. (7), we extracted an explicit factor  $m_l/M$  out of the amplitudes, which reflects the fact that for a vector interaction (such as in QED), the lepton helicity flip amplitude vanishes when  $m_l \rightarrow 0$ . In the Born approximation, one obtains :

$$\begin{aligned} \tilde{G}_M^{Born}(\nu, Q^2) &= G_M(Q^2), \\ \tilde{F}_2^{Born}(\nu, Q^2) &= F_2(Q^2), \\ \tilde{F}_{3,4,5,6}^{Born}(\nu, Q^2) &= 0, \end{aligned} \quad (8)$$

where  $G_M(Q^2)$  and  $F_2(Q^2)$  are the magnetic and Pauli form factors respectively. Since  $\tilde{F}_3$ ,  $\tilde{F}_4$ ,  $\tilde{F}_5$ ,  $\tilde{F}_6$ , and the phases of  $\tilde{G}_M$  and  $\tilde{F}_2$  vanish in the Born approximation, they must originate from processes involving at least the exchange of two photons. Relative to the factor  $e^2$  introduced in Eq. (6), we see that these are at least of order  $e^2$ . For convenience, we trade the invariant  $\tilde{F}_2$  for  $\tilde{G}_E$ , defined as :

$$\tilde{G}_E \equiv \tilde{G}_M - (1 + \tau) \tilde{F}_2, \quad (9)$$

In the Born approximation  $\tilde{G}_E$  reduces to the electric form factor  $G_E(Q^2)$ . For a beam polarized normal to the scattering plane, we can define a single spin asymmetry,

$$B_n = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}, \quad (10)$$

where  $\sigma_{\uparrow}$  ( $\sigma_{\downarrow}$ ) denotes the cross section for an unpolarized target and for a lepton beam spin parallel (anti-parallel) to the normal polarization vector, defined as :

$$S_n^\mu = (0, \vec{S}_n), \quad \vec{S}_n \equiv (\vec{k} \times \vec{k}') / |\vec{k} \times \vec{k}'|. \quad (11)$$

We refer to this asymmetry as the beam normal spin asymmetry ( $B_n$ ). Its leading non-vanishing contribution is linear in the lepton mass. Furthermore,  $B_n$  vanishes in the Born approximation, and is therefore of order  $e^2$ . Keeping only the leading term of order  $e^2$ ,  $B_n$  arises from an interference between the one- and two-photon exchange amplitudes. In terms of the invariants of Eqs. (6,7),  $B_n$  is given by :

$$\begin{aligned} B_n = & \frac{2m_l}{Q} \sqrt{2\varepsilon(1-\varepsilon)} \sqrt{1 + \frac{1}{\tau}} \left( G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \right)^{-1} \\ & \times \left\{ -\tau G_M \mathcal{I} \left( \tilde{F}_3 + \frac{1}{1+\tau} \frac{\nu}{M^2} \tilde{F}_5 \right) - G_E \mathcal{I} \left( \tilde{F}_4 + \frac{1}{1+\tau} \frac{\nu}{M^2} \tilde{F}_5 \right) \right\} \\ & + \mathcal{O}(e^4), \end{aligned} \quad (12)$$

where  $\mathcal{I}$  denotes the imaginary part.

### 3 Imaginary part of elastic lepton-quark scattering

To evaluate the two photon exchange amplitudes, we decompose the lower blob in Fig. 1 into a part where the intermediate state is a nucleon, which we call the nucleon pole part, and the rest which we call the inelastic part. The nucleon pole part is exactly calculable since it involves only the on shell form factors and we discuss only the inelastic part which we model by the handbag diagram (see Fig. 2) where the photon scatters off a quark which is approximately on the mass shell. The first step is the evaluation of the elastic lepton-quark scattering :

$$l(k) + q(p_q) \rightarrow l(k') + q(p'_q), \quad (13)$$

which involves two independent kinematical invariants :  $\hat{s} \equiv (k + p_q)^2$  and  $Q^2 = -(k - k')^2$ . For further use, we also introduce the crossing variable  $\hat{u} \equiv (k - p'_q)^2$ , which satisfies  $\hat{s} + \hat{u} = Q^2$  (for massless quarks). The  $T$ -matrix for the lepton-quark hard scattering process can be written as :

$$H_{h'h,\lambda}^{hard} = \frac{(e e_q)^2}{Q^2} \left\{ \bar{u}(k', h') \gamma_\mu u(k, h) \cdot \bar{u}(p'_q, \lambda) \left( \tilde{f}_1 \gamma^\mu + \tilde{f}_3 \gamma \cdot K P_q^\mu \right) u(p_q, \lambda) \right\}$$

$$+ m_l \tilde{f}_5 \bar{u}(k', h') u(k, h) \cdot \bar{u}(p'_q, \lambda) \gamma \cdot K u(p_q, \lambda) \Big\}, \quad (14)$$

with  $P_q \equiv (p_q + p'_q)/2$ , where  $e_q$  is the charge of the quark in units of  $e$ , and where  $u(p_q, \lambda)$  and  $u(p'_q, \lambda)$  are the quark spinors with quark helicity  $\lambda = \pm 1/2$ , which is conserved in the hard scattering process. In Eq. (14),  $\tilde{f}_1, \tilde{f}_3$  and  $\tilde{f}_5$  are the invariant amplitudes for the scattering of leptons off massless quarks ( $m_q = 0$ ). They are the analogues of the invariants introduced in Eqs. (6, 7) for the nucleon except that, for obvious reasons, we do not introduce powers of the quark mass to make them dimensionless. In the one-photon exchange approximation  $\tilde{f}_1 \rightarrow 1$ , and  $\tilde{f}_3 \rightarrow 0, \tilde{f}_5 \rightarrow 0$ . Note that for massless quarks, quark helicity conservation leads to the absence of analogues of  $\tilde{F}_2, \tilde{F}_4$  and  $\tilde{F}_6$ . To calculate the beam normal spin asymmetry  $B_n$ , we need the corresponding expressions for the imaginary parts of  $\tilde{f}_1, \tilde{f}_3$  and  $\tilde{f}_5$ . These imaginary parts originate solely from the direct two-photon exchange box diagram on the quark level. The amplitudes  $\tilde{f}_1$  and  $\tilde{f}_3$ , which conserve the lepton helicity, were already calculated in Ref. [8]. It was shown in that work that the amplitude  $\tilde{f}_1$  can be separated into a soft and hard part, i.e.  $\tilde{f}_1 = \tilde{f}_1^{soft} + \tilde{f}_1^{hard}$ . The soft part corresponds with the situation where one of the photons in Fig. 1 carries zero four-momentum, and is obtained by replacing the other photon's four-momentum by  $q$  in both numerator and denominator of the loop integral. This yields (for  $\hat{s} > 0$  and  $\hat{u} < 0$ ) the expressions [8] :

$$\mathcal{I}(\tilde{f}_1^{soft}) = -\frac{e^2}{4\pi} \ln\left(\frac{\lambda^2}{\hat{s}}\right), \quad (15)$$

$$\mathcal{I}(\tilde{f}_1^{hard}) = -\frac{e^2}{4\pi} \left\{ \frac{Q^2}{2\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + \frac{1}{2} \right\}, \quad (16)$$

$$\mathcal{I}(\tilde{f}_3) = -\frac{e^2}{4\pi} \frac{1}{\hat{u}} \left\{ \frac{\hat{s} - \hat{u}}{\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + 1 \right\}. \quad (17)$$

Note that the IR divergent part in Eq. (15), proportional to the fictitious photon mass  $\lambda^2$ , does not contribute when calculating physical observables as it just corresponds to the lowest order calculation of the Coulomb phase of the amplitude.

Analogously to Eqs. (15, 16, 17), we can calculate the imaginary part of the amplitude  $\tilde{f}_5$  and obtain (for  $\hat{s} > 0$  and  $\hat{u} < 0$ ) :

$$\mathcal{I}(\tilde{f}_5)_{2\gamma} = -\frac{e^2}{4\pi} \frac{Q^2}{\hat{u}} \left\{ -\frac{1}{\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) - \frac{1}{\hat{s}} \right\}. \quad (18)$$

As a useful check we use the previous results to evaluate the beam normal spin asymmetry  $B_n$  on a quark. In the limit of massless quarks,  $B_n$  is given (for  $e_q = +1$ ) by :

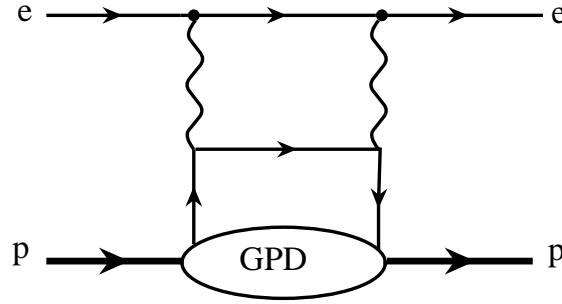


Fig. 2. Handbag contribution to the two-photon exchange amplitude entering the normal beam asymmetry for elastic lepton-nucleon scattering. The lower blob represents the GPDs of the nucleon.

$$B_n = \frac{m_l}{Q} \sqrt{2\varepsilon(1-\varepsilon)} \frac{1}{2} \left\{ -Q^2 \mathcal{I}(\tilde{f}_3)_{2\gamma} - (\hat{s} - \hat{u}) \mathcal{I}(\tilde{f}_5)_{2\gamma} \right\}. \quad (19)$$

Using the lepton-quark amplitudes of Eqs. (17,18), this yields :

$$B_n = \frac{e^2}{4\pi} \frac{m_l}{Q} \sqrt{2\varepsilon(1-\varepsilon)} \frac{Q^2}{2\hat{s}} = \frac{e^2}{4\pi} \frac{m_l}{Q} \sqrt{\frac{-\hat{u}}{\hat{s}}} \frac{Q^4}{\hat{s}^2 + \hat{u}^2}, \quad (20)$$

where the last step has been obtained by using  $\varepsilon = -2\hat{s}\hat{u}/(\hat{s}^2 + \hat{u}^2)$ . The expression of Eq. (20) agrees with the well known result for the beam normal SSA for  $e^+\mu^- \rightarrow e^-\mu^-$  derived long time ago in Ref. [9] (see Eq. (15) in that work, by taking the muon mass equal to zero).

#### 4 Imaginary part of elastic lepton-nucleon scattering in terms of generalized parton distributions

Having discussed the two-photon exchange amplitude on the quark, we calculate the corresponding amplitudes on the nucleon, as is shown in Fig. 2. We follow Ref. [8] and calculate the amplitudes in the kinematical regime where  $s, -u$  and  $Q^2$  are large compared to a hadronic scale ( $s, -u, Q^2 \gg M^2$ ) as a convolution between a hard scattering electron-quark amplitude and a soft matrix element on the nucleon. For this kinematical regime, it is convenient to choose a frame where  $q^+ = 0$ , as in [19], where we introduce light-cone variables  $a^\pm = (a^0 \pm a^3)/\sqrt{2}$  and choose the  $z$ -axis along the direction of  $P^3$  (so that  $P$  has a large  $+$  component). The lepton  $+$  momentum fractions are given by  $\eta = K^+/P^+$ , with



$$\eta = (s - u - 2\sqrt{M^4 - su}) / (Q^2 + 4M^2).$$

In the frame  $q^+ = 0$ , the parton light-cone momentum fractions are then defined as  $x = p_q^+ / P^+ = p_q'^+ / P^+$ . The active partons, on which the hard scattering takes place, are approximately on-shell, and their intrinsic transverse momenta (defined in a frame where the hadron has zero transverse momentum) are small and can be neglected when evaluating the hard scattering process. The Mandelstam variables for the process (13) on the quark, which enter in the evaluation of the hard scattering amplitude, are given by :

$$\hat{s} = \frac{(x + \eta)^2}{4x\eta} Q^2, \quad \hat{u} = -\frac{(x - \eta)^2}{4x\eta} Q^2. \quad (21)$$

The helicity amplitudes for elastic electron-nucleon scattering in the kinematical regime where  $s, -u, Q^2 \gg M^2$ , can be expressed as [8] :

$$\begin{aligned} T_{h'\lambda'_N, h\lambda_N}^{hard} &= \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} [H_{h'h, +\frac{1}{2}}^{hard} + H_{h'h, -\frac{1}{2}}^{hard}] \\ &\quad \times \left\{ [H^q(x, 0, q^2) + E^q(x, 0, q^2)] \frac{1}{2} \bar{u}(p', \lambda'_N) \gamma \cdot n u(p, \lambda_N) \right. \\ &\quad \left. - E^q(x, 0, q^2) \frac{1}{2M} \bar{u}(p', \lambda'_N) u(p, \lambda_N) \right] \\ &+ \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} [H_{h'h, +\frac{1}{2}}^{hard} - H_{h'h, -\frac{1}{2}}^{hard}] \\ &\quad \times \text{sgn}(x) \tilde{H}^q(x, 0, q^2) \frac{1}{2} \bar{u}(p', \lambda'_N) \gamma \cdot n \gamma_5 u(p, \lambda_N), \end{aligned} \quad (22)$$

where  $H_{h'h, \lambda}^{hard}$  are the hard scattering amplitudes, and  $n^\mu$  is a Sudakov four-vector ( $n^2 = 0$ ), which is given by :

$$n^\mu = 2/(\sqrt{M^4 - su}) \{-\eta P^\mu + K^\mu\}. \quad (23)$$

Furthermore in Eq. (22),  $H^q, E^q, \tilde{H}^q$  are the Generalized Parton Distributions (GPDs) for a quark  $q$  in the nucleon (for a review, see e.g. Ref. [22]). The hard scattering amplitudes  $H_{h'h, \lambda}^{hard}$  in Eq. (22) can be expressed in terms of  $\tilde{f}_1^{hard}, \tilde{f}_3, \tilde{f}_5$  using Eq. (14) as :

$$\begin{aligned} \frac{1}{2} [H_{h'h, +\frac{1}{2}}^{hard} + H_{h'h, -\frac{1}{2}}^{hard}] &= \frac{(e e_q)^2}{Q^2} \left\{ \delta_{h', h} [\tilde{f}_1^{hard} (\hat{s} - \hat{u}) - \tilde{f}_3 \hat{s} \hat{u}] \right. \\ &\quad \left. - \delta_{h', -h} (2h) m_l \sqrt{Q^2} \sqrt{-\hat{s} \hat{u}} \left[ \frac{2}{\hat{s}} \tilde{f}_1^{hard} + \tilde{f}_3 + \tilde{f}_5 \right] \right\}, \end{aligned} \quad (24)$$

$$\frac{1}{2} \left[ H_{h'h,+\frac{1}{2}}^{hard} - H_{h'h,-\frac{1}{2}}^{hard} \right] = \frac{(e e_q)^2}{Q^2} \delta_{h',h} (2h) \tilde{f}_1^{hard} Q^2, \quad (25)$$

with  $\hat{s}$  and  $\hat{u}$  according to Eq. (21).

To extract  $\tilde{F}_3, \tilde{F}_4, \tilde{F}_5$ , we first need to express them in terms of the electron-nucleon helicity amplitudes, which is done in Appendix A. Using Eq. (22) and Eqs. (A.2)-(A.4), we can finally express  $\tilde{F}_3, \tilde{F}_4, \tilde{F}_5$  (after some algebra) in terms of the nucleon GPDs as :

$$\tilde{F}_3 = \frac{M^2}{\nu} \left( \frac{1+\varepsilon}{2\varepsilon} \right) (A - C), \quad (26)$$

$$\begin{aligned} \tilde{F}_4 = & \frac{1}{1+\tau} \left\{ \sqrt{\frac{1+\varepsilon}{2\varepsilon}} \left[ \frac{s+M^2}{s-M^2} \sqrt{\frac{1+\varepsilon}{2\varepsilon}} A - A' \right] - \left[ \frac{s+M^2}{s-M^2} \sqrt{\frac{1+\varepsilon}{2\varepsilon}} B - B' \right] \right\} \\ & - \frac{M^2}{\nu} \left( \frac{1+\varepsilon}{2\varepsilon} \right) C, \end{aligned} \quad (27)$$

$$\begin{aligned} \tilde{F}_5 = & -\frac{M^2}{\nu} \sqrt{\frac{1+\varepsilon}{2\varepsilon}} \left[ \frac{s+M^2}{s-M^2} \sqrt{\frac{1+\varepsilon}{2\varepsilon}} A - A' \right] \\ & + \left( \frac{M^2}{\nu} \right)^2 \left( \frac{1+\varepsilon}{2\varepsilon} \right) (1+\tau) C, \end{aligned} \quad (28)$$

where we introduced the integrals containing the GPDs :

$$\begin{aligned} A &\equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s} \hat{u} \tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q + E^q), \\ B &\equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s} \hat{u} \tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q), \\ C &\equiv \int_{-1}^1 \frac{dx}{x} \tilde{f}_1^{hard} \text{sgn}(x) \sum_q e_q^2 \tilde{H}^q, \\ A' &\equiv \int_{-1}^1 \frac{dx}{x} \frac{\sqrt{-\hat{s} \hat{u}}}{2} \left[ \frac{2}{\hat{s}} \tilde{f}_1^{hard} + \tilde{f}_3 + \tilde{f}_5 \right] \sum_q e_q^2 (H^q + E^q), \\ B' &\equiv \int_{-1}^1 \frac{dx}{x} \frac{\sqrt{-\hat{s} \hat{u}}}{2} \left[ \frac{2}{\hat{s}} \tilde{f}_1^{hard} + \tilde{f}_3 + \tilde{f}_5 \right] \sum_q e_q^2 (H^q - \tau E^q). \end{aligned} \quad (29)$$

The expression for  $\tilde{F}_3$  and the quantities  $A, B, C$  have been given previously in Ref. [8]. Eqs. (26-28) reduce to the partonic amplitudes in the limit  $M \rightarrow 0$  by considering a quark target, for which the GPDs are given by :  $H^q \rightarrow \delta(1-x)$ ,  $E^q \rightarrow 0$ , and  $\tilde{H}^q \rightarrow \delta(1-x)$ . In this limit, and using the identity  $-\hat{s} \hat{u} = 4\nu^2 \left( \frac{2\varepsilon}{1+\varepsilon} \right)$ , we then easily recover that  $\tilde{F}_3/M^2 \rightarrow e_q^2 \tilde{f}_3$ ,  $\tilde{F}_4/M \rightarrow 0$ , and

$$\tilde{F}_5/M^2 \rightarrow e_q^2 \tilde{f}_5.$$

Inserting the above expressions of Eqs. (26-28) for  $\tilde{F}_3$ ,  $\tilde{F}_4$  and  $\tilde{F}_5$  into Eq. (12), we can work out  $B_n$ , which becomes :

$$\begin{aligned} B_n = & \frac{2m_l}{Q} \sqrt{2\varepsilon(1-\varepsilon)} \left( G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \right)^{-1} \\ & \times \left\{ G_M \left[ \frac{\sqrt{1+\varepsilon}\sqrt{\tau}}{\sqrt{1+\varepsilon}\sqrt{1+\tau} + \sqrt{1-\varepsilon}\sqrt{\tau}} \mathcal{I}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}} \sqrt{\frac{\tau}{1+\tau}} \mathcal{I}(A') \right] \right. \\ & \left. + \frac{1}{\tau} G_E \left[ \frac{\sqrt{1+\varepsilon}\sqrt{\tau} + \sqrt{1-\varepsilon}\sqrt{1+\tau}}{\sqrt{1+\varepsilon}\sqrt{1+\tau} + \sqrt{1-\varepsilon}\sqrt{\tau}} \sqrt{\frac{1+\varepsilon}{2\varepsilon}} \mathcal{I}(B) - \sqrt{\frac{\tau}{1+\tau}} \mathcal{I}(B') \right] \right\} \\ & + \mathcal{O}(e^4). \end{aligned} \quad (30)$$

Note that the soft part of  $\tilde{f}_1$  (Eq. (15)) only enters in the amplitudes  $\tilde{G}_M$  and  $\tilde{F}_2$  as shown in Ref. [8], and does not enter into  $B_n$  which is IR finite. One sees from Eq. (30) that  $B_n$  contains two terms : a first one in which the magnetic form factor  $G_M$  is multiplied by a “magnetic GPD” ( $H^q + E^q$ ), and a second one in which the electric form factor  $G_E$  is multiplied by an “electric GPD” ( $H^q - \tau E^q$ ). Furthermore, one notices that  $B_n$  does not depend upon the GPD  $\tilde{H}$ .

To estimate the two-photon exchange amplitudes according to Eqs. (26,27,28), we need to specify a model for the GPDs. As  $B_n$  does not depend upon  $\tilde{H}^q$ , we only need to specify the model for the GPDs  $H^q$  and  $E^q$ . Following Ref. [20], we use an unfactorized (valence) model in  $x$  and  $t$  for the GPD  $H$  as :

$$H^q(x, 0, q^2) = q_v(x) \exp \left( -\frac{(1-x)Q^2}{4x\sigma} \right), \quad (31)$$

where  $q_v(x)$  is the valence quark distribution. In the following estimates we take the unpolarized parton distributions at input scale  $Q_0^2 = 1 \text{ GeV}^2$  from the MRST2002 global NNLO fit [23] as :

$$\begin{aligned} u_v &= 0.262 x^{-0.69} (1-x)^{3.50} \left( 1 + 3.83 x^{0.5} + 37.65 x \right), \\ d_v &= 0.061 x^{-0.65} (1-x)^{4.03} \left( 1 + 49.05 x^{0.5} + 8.65 x \right). \end{aligned}$$

For the GPD  $E$ , whose forward limit is unknown, we adopt a valence parametrization multiplied with  $(1-x)^2$  to be consistent with the  $x \rightarrow 1$  limit [24]. This yields :

$$E^q(x, 0, q^2) = \frac{\kappa^q}{N^q} (1-x)^2 q_v(x) \exp \left( -\frac{(1-x)Q^2}{4x\sigma} \right), \quad (32)$$

where the normalization factors  $N^u, N^d$  are chosen in such a way that the first moments of  $E^u$  and  $E^d$  at  $Q^2 = 0$  yield the anomalous magnetic moments  $\kappa^u = 2\kappa^p + \kappa^n = 1.673$  and  $\kappa^d = \kappa^p + 2\kappa^n = -2.033$  respectively. Furthermore, the parameter  $\sigma$  in Eqs. (31,32) can be related to the average transverse momentum of the quarks inside the nucleon as  $\sigma = 5 \langle k_\perp^2 \rangle$ . Its value has been estimated in Ref. [21] as :  $\sigma \simeq 0.8 \text{ GeV}^2$ , which we will adopt in the following calculations.

## 5 Results and discussion

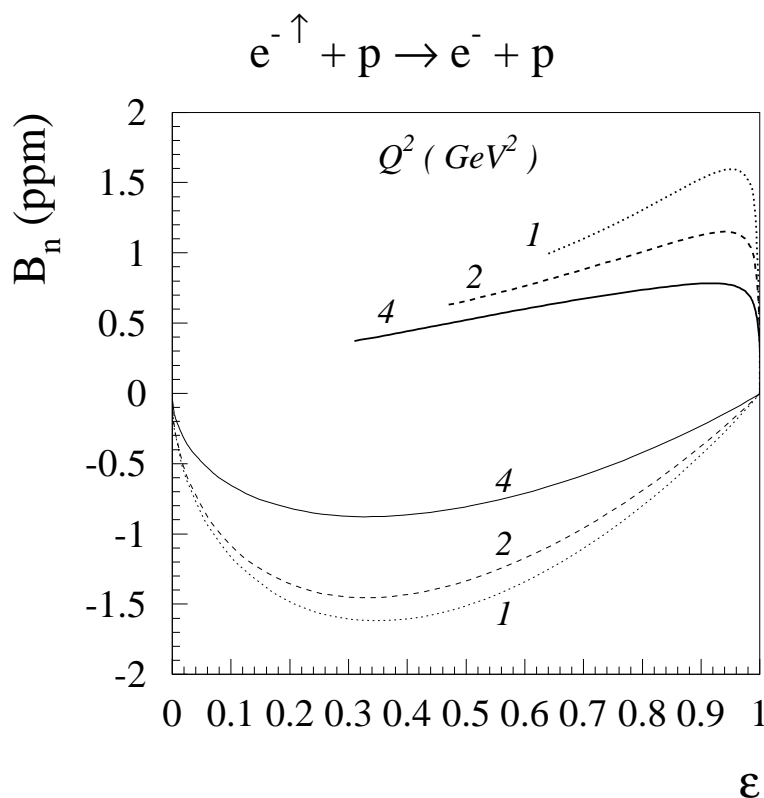


Fig. 3. Beam normal spin asymmetry for elastic  $e^-p$  scattering as function of  $\epsilon$  at different values of  $Q^2$  as indicated on the figure. The upper thick curves ( $B_n > 0$ ) are the GPD calculations for the kinematical range where  $s, -u > M^2$ . For comparison, the nucleon pole contribution is also displayed : lower thin curves ( $B_n < 0$ ).

In Fig. 3, we show our results for  $B_n$  at several values of  $Q^2$  (for  $Q^2 > 1 \text{ GeV}^2$ ) as a function of  $\epsilon$ . One sees that the handbag calculation for the inelastic part to  $B_n$  yields asymmetries which are forward peaked and are in the range of +1 ppm to +1.5 ppm. For comparison, the nucleon pole contribution is also displayed. For the proton form factors, we use the  $G_{Ep}/G_{Mp}$  ratio as ex-

tracted from the polarization transfer experiments [2]. For  $G_{Mp}$ , we adopt the parametrization of Ref. [25]. One sees from Fig. 3 that the nucleon pole contribution to  $B_n$  has a sign opposite to the inelastic one and also has a different energy dependence. Whereas the inelastic part peaks at large  $\varepsilon$  (forward direction), the nucleon pole contribution is small in the forward region and reaches its maximum at  $\varepsilon$  values around 0.3. As this nucleon pole contribution is well known, one can always add it to the data to extract the inelastic part from experiment (in analogy with what is usually done to extract the inelastic part of moments of nucleon structure functions).

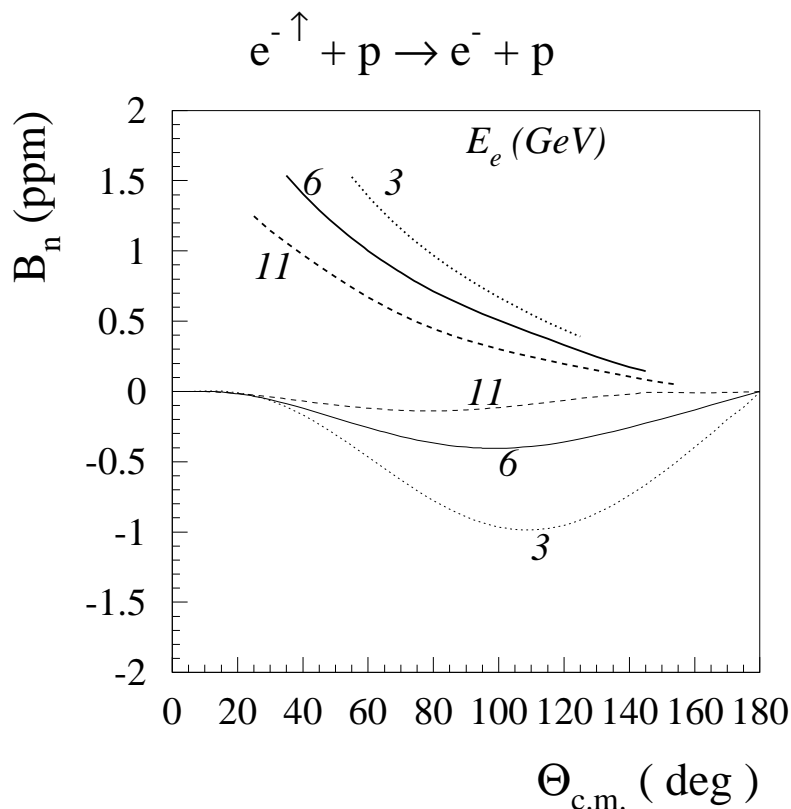


Fig. 4. Beam normal spin asymmetry for elastic  $e^-p$  scattering as function of the  $c.m.$  electron scattering angle at different electron beam energies as indicated on the figure. The upper thick curves ( $B_n > 0$ ) are the GPD calculations for the kinematical range where  $-u, Q^2 > M^2$ . For comparison, the nucleon pole contribution is also displayed : lower thin curves ( $B_n < 0$ ).

In Fig. 4, we display our results for  $B_n$  for elastic  $e^{-\uparrow}p \rightarrow e^-p$  scattering at fixed beam energy as function of the elastic scattering  $c.m.$  angle in the energy range accessible at Jefferson Lab. One clearly sees that the forward angular range is a favorable region to get information on the inelastic part of  $B_n$ . As we have discussed before, this inelastic part is a direct measure of the imaginary part of the two-photon exchange amplitude. In the handbag calculation considered in this work, the real and imaginary parts are related through a

dispersion relation. Hence a measurement of the inelastic contribution to  $B_n$  would yield a useful cross-check on the handbag estimate for the real part, which was found crucial to resolve the discrepancy between Rosenbluth and polarization transfer experiments on the proton [8].

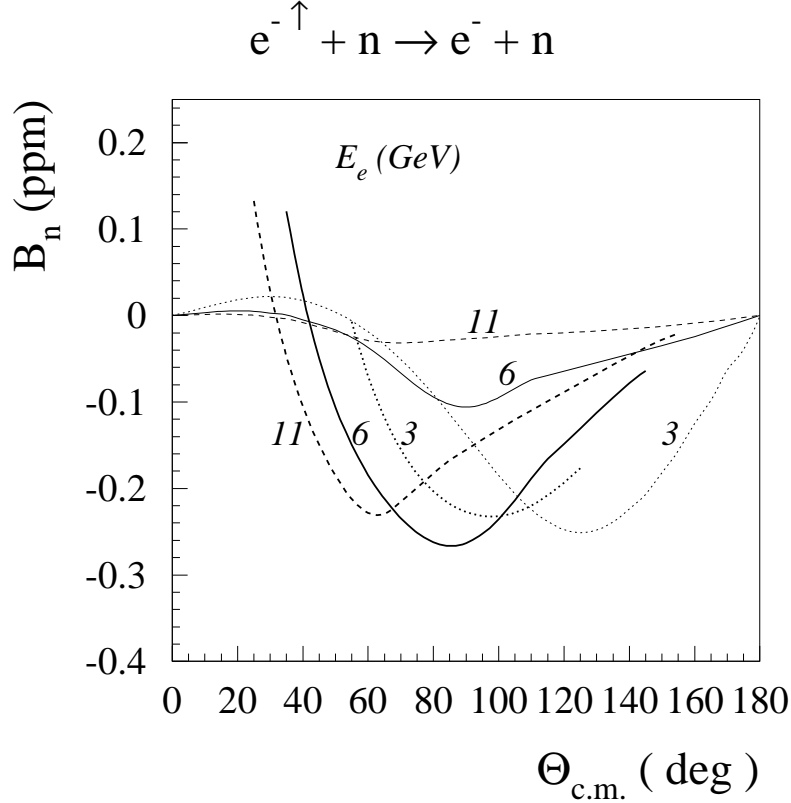


Fig. 5. Beam normal spin asymmetry for elastic  $e^-n$  scattering as function of the *c.m.* electron scattering angle at different electron beam energies as indicated on the figure. The thick curves are the GPD calculations for the kinematical range where  $-u, Q^2 > M^2$ . For comparison, the nucleon pole contribution is also displayed by the thin curves.

Within the handbag model described above, we also get a prediction for  $B_n$  on the neutron. The two-photon exchange amplitude on the neutron is obtained by interchanging up and down-quark distributions in the corresponding expression for the proton. We show the neutron result for  $B_n$  in Fig. 5. One sees that the inelastic part of  $B_n$  for the neutron is much smaller than its proton counterpart. One can easily understand this from the expression of Eq. (12) for  $B_n$ , which involves a term proportional to  $G_M$  and a term proportional to  $G_E$ . For the proton case, both terms add with the same sign, whereas for the neutron case,  $G_M$  has the opposite sign. This results in a partial cancellation in  $B_n$ . Furthermore, one sees from Fig. 5 that  $B_n$  changes sign when going to the backward region (where the term proportional to  $G_M$  dominates). In Fig. 5, we also show the nucleon pole contribution to  $B_n$  for the  $e^{-\uparrow}n \rightarrow e^-n$

process. For the neutron form factors, we use the recent  $G_{En}$  parametrization of Ref. [26], and the  $G_{Mn}$  parametrization of Ref. [27]. It is seen that also the nucleon pole contribution to  $B_n$  for the neutron is suppressed, and is largely negligible compared to the inelastic one in the forward angular range.

## 6 Conclusions

In summary, we have developed in this work the formalism for elastic lepton-nucleon scattering with a lepton helicity flip, beyond the one-photon exchange approximation. We have shown that the imaginary part of the two-photon exchange amplitude can be accessed through the single spin asymmetry for a lepton beam polarized normal to the scattering plane. We provided an estimate for this normal beam spin asymmetry  $B_n$  in a partonic framework, used before to evaluate the lepton helicity conserving amplitudes. In our calculation, the normal beam spin asymmetry at large momentum transfer is estimated through the scattering off a parton, which is embedded in the nucleon through a generalized parton distribution. Using phenomenological parametrizations for the GPDs, we found that for the proton,  $B_n$  yields values around +1 ppm to +1.5 ppm in the few GeV beam energy range. In particular, we found that the forward angular range for  $e^-p \rightarrow e^-p$  scattering is a favorable region to get information on the inelastic part of  $B_n$ . Because in the handbag calculation considered here, real and imaginary parts are linked, a direct measurement of  $B_n$  may yield a valuable cross-check of the estimate for the real part, which was found crucial in understanding the unpolarized cross section data for  $e^-p \rightarrow e^-p$  at large momentum transfer. A measurement of  $B_n$  can be performed by the same experiments that are set up to measure parity violation in  $\vec{e}^-p \rightarrow e^-p$  scattering by choosing a normal polarization for the electron beam instead. This may open up a new experimental front to access the two-photon exchange amplitudes in elastic electron-nucleon scattering.

## Acknowledgments

This work was supported by the Italian MIUR and INFN, by the French Commissariat à l’Energie Atomique (CEA), and by the U.S. Department of Energy under contracts DE-FG02-04ER41302 and DE-AC05-84ER40150.

## A Relations between helicity amplitudes and invariant amplitudes for elastic electron-nucleon scattering

In this appendix we express the amplitudes  $\tilde{F}_3$ ,  $\tilde{F}_4$  and  $\tilde{F}_5$  which enter into the beam normal spin asymmetry of Eq. (12) in terms of the helicity amplitudes for elastic lepton-nucleon scattering. For convenience, we introduce for the six independent helicity amplitudes of Eqs. (6, 7) (in the lepton-nucleon *c.m.* system) the shorthand notation :

$$\begin{aligned} T_1 &\equiv T_{h'=\frac{1}{2}, \lambda'_N=\frac{1}{2}, h=\frac{1}{2}, \lambda_N=\frac{1}{2}}, & T_4 &\equiv T_{h'=-\frac{1}{2}, \lambda'_N=\frac{1}{2}, h=\frac{1}{2}, \lambda_N=\frac{1}{2}}, \\ T_2 &\equiv T_{h'=\frac{1}{2}, \lambda'_N=-\frac{1}{2}, h=\frac{1}{2}, \lambda_N=\frac{1}{2}}, & T_5 &\equiv T_{h'=-\frac{1}{2}, \lambda'_N=-\frac{1}{2}, h=\frac{1}{2}, \lambda_N=\frac{1}{2}}, \\ T_3 &\equiv T_{h'=\frac{1}{2}, \lambda'_N=-\frac{1}{2}, h=\frac{1}{2}, \lambda_N=-\frac{1}{2}}, & T_6 &\equiv T_{h'=-\frac{1}{2}, \lambda'_N=\frac{1}{2}, h=\frac{1}{2}, \lambda_N=-\frac{1}{2}}. \end{aligned} \quad (\text{A.1})$$

The amplitudes  $\tilde{F}_3$ ,  $\tilde{F}_4$  and  $\tilde{F}_5$  can be expressed in terms of the *c.m.* helicity amplitudes of Eq. (A.1) as :

$$e^2 \frac{\tilde{F}_3}{M^2} = \frac{1}{(s-M^2)} \left\{ -T_1 + \frac{2M\sqrt{Q^2}}{\sqrt{M^4-su}} T_2 + \left( \frac{(s^2-M^4)-M^2(s-u)}{(M^4-su)} \right) T_3 \right\}, \quad (\text{A.2})$$

$$\begin{aligned} e^2 \frac{\tilde{F}_4}{M} &= \frac{M}{(s-M^2)} \left[ -T_1 + \frac{(s+M^2)}{M} \frac{\sqrt{Q^2}}{\sqrt{M^4-su}} T_2 + \left( \frac{(s^2-M^4)-M^2(s-u)}{(M^4-su)} \right) T_3 \right] \\ &\quad + \frac{\sqrt{Q^2}}{\sqrt{M^4-su}} \frac{M}{m_l} T_4 + \frac{1}{2m_l} (T_5 - T_6), \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} e^2 \frac{\tilde{F}_5}{M^2} &= -\frac{2M^2}{(s-M^2)^2} \left[ -T_1 + \frac{(s+M^2)}{M} \frac{\sqrt{Q^2}}{\sqrt{M^4-su}} T_2 + \left( \frac{s(s^2+su-2M^4)-M^4(s-u)}{2M^2(M^4-su)} \right) T_3 \right] \\ &\quad - \frac{(s+M^2)}{(s-M^2)} \frac{\sqrt{Q^2}}{\sqrt{M^4-su}} \frac{1}{m_l} T_4 - \frac{1}{(s-M^2)} \frac{M}{m_l} (T_5 - T_6), \end{aligned} \quad (\text{A.4})$$

where we keep only the leading term when taking  $m_l \rightarrow 0$ .

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